JP Morgan Startup Survival Rates
The Continuous-Time Survival Rate Curve

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The startup survival rates from Part I are in discrete-time (annual increments). In this white paper we will map the JP Morgan survival rates (the corresponding failure rate is equal to one minus the survival rate) to a continuous-time curve. To that end we will work through the following hypothetical problem from Part I...

Our Hypothetical Problem

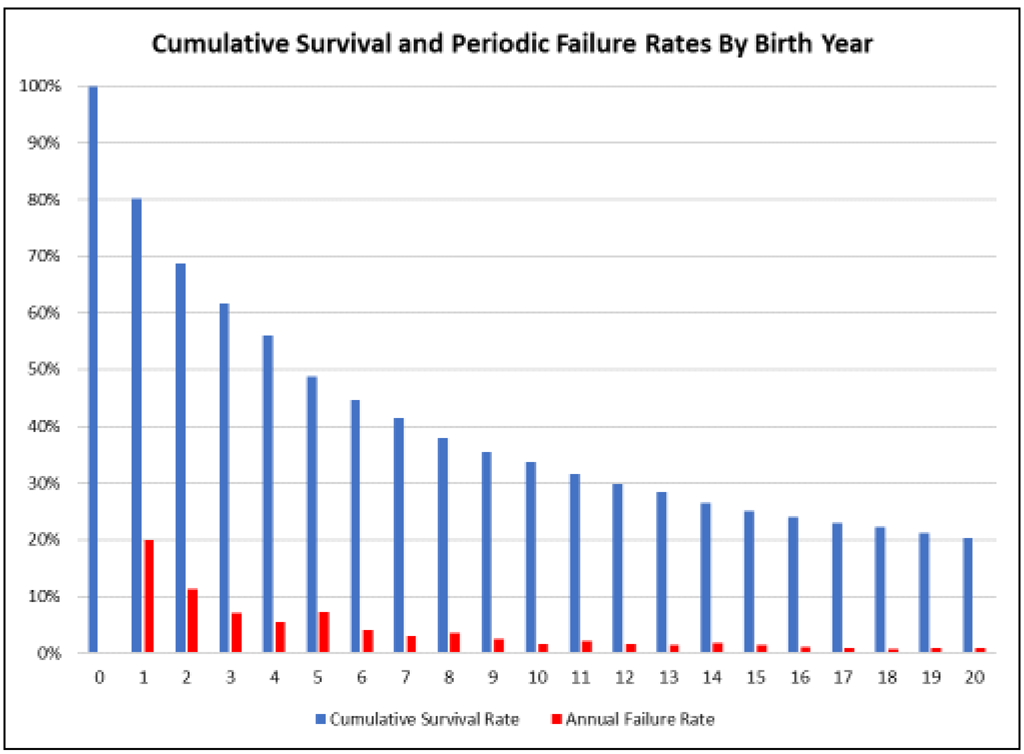
We are given the following startup survival and failure rates by birth year...

Source: https://www.jpmorganchase.com/institute/research/small-business/small-business-dashboard/longevity

Table 1: JP Morgan Data

Birth Year	Survival Rate	Failure Rate
0	1.0000	–
1	0.8010	0.1990
2	0.6870	0.1140
3	0.6160	0.0710
4	0.5600	0.0560
5	0.4880	0.0720
6	0.4470	0.0410
7	0.4160	0.0310
8	0.3800	0.0360
9	0.3550	0.0250
10	0.3380	0.0170
11	0.3160	0.0220
12	0.2990	0.0170
13	0.2840	0.0150
14	0.2660	0.0180
15	0.2510	0.0150
16	0.2400	0.0110
17	0.2300	0.0100
18	0.2220	0.0080
19	0.2130	0.0090
20	0.2040	0.0090
Total	–	0.7960

Table 2: Survival And Failure Rate Graph By Birth Year In Discrete-Time



Note that in the graph above survival rates are approximately Exponentially-distributed (i.e. the first derivative is negative (survival rates decrease over time) and the second derivative is positive (survival rates decrease over time but at a decreasing rate). The Exponential distribution is a one parameter probability distribution.

Question: Given that ABC Company has survived to year 3.25, what is the probability that ABC company will survive until year 7.75?

Mapping Survival Rates to an Exponential Distribution

We will define the function $S(t)$ to be the survival function at time t, which is the probability that a startup company survives over the time interval $[0, t]$ where time zero is the time in years that the company began operations. If the survival rate is an Exponentially-distributed random variable then the equation for the expected survival rate at time t is... [1]

$$S(t) = \text{Exp}\left\{ -\lambda\, t \right\} \text{ ...where... } \lambda = \text{the hazard rate } \text{ ...and... } 0 \leq \lambda \leq 1 \tag{1}$$

We want to map the survival rates in Table 1 above to the Exponential distribution. If we define N to be sample size then the equation for the parameter λ via a Maximum Log Likelihood MLL) estimation methodology is... [2]

$$\lambda = \left[\frac{1}{N} \sum_{i=1}^{N} t_i \right]^{-1} \tag{2}$$

Using the data in Table 1 above we will define the following parameters...

$$N = 21 \text{ ...and... } \sum_{i=1}^{N} t_i = 210 \tag{3}$$

Using Equations (2) and (3) above the MLL estimate of the hazard rate parameter λ is...

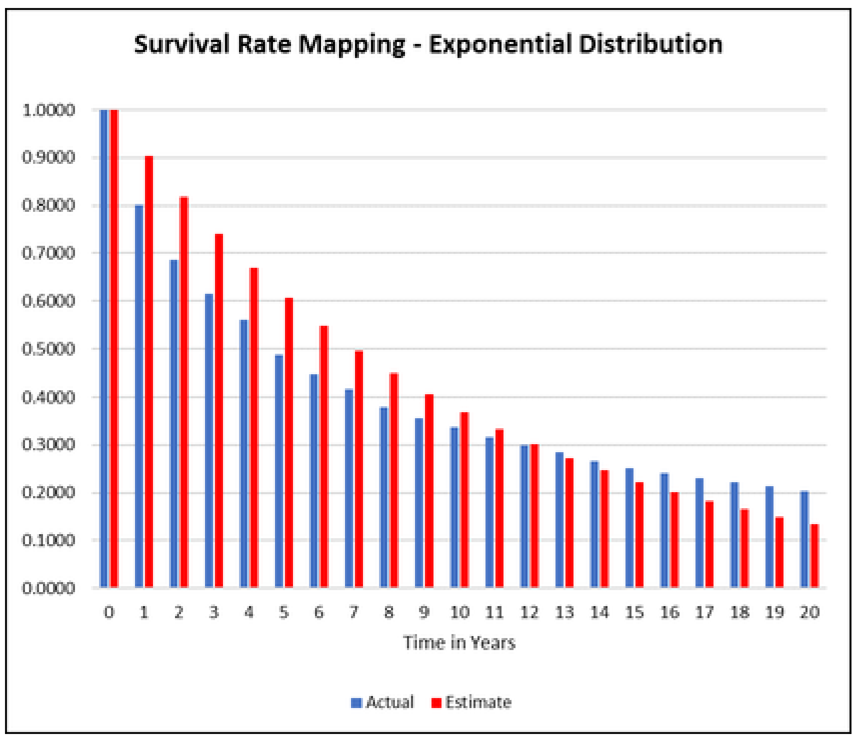
$$\lambda = \left[\frac{1}{21} \times 210 \right]^{-1} = 0.1000 \tag{4}$$

Using Equations (1) and (4) above and the data in Table 1 above the graph of actual survival rates versus estimated survival rates via an Exponential distribution is...

Table 3: Survival Rates

Birth Year	Actual Rate	Estimate Rate	Est-Act Residual
0	1.0000	1.0000	–
1	0.8010	0.9048	0.1038
2	0.6870	0.8187	0.1317
3	0.6160	0.7408	0.1248
4	0.5600	0.6703	0.1103
5	0.4880	0.6065	0.1185
6	0.4470	0.5488	0.1018
7	0.4160	0.4966	0.0806
8	0.3800	0.4493	0.0693
9	0.3550	0.4066	0.0516
10	0.3380	0.3679	0.0299
11	0.3160	0.3329	0.0169
12	0.2990	0.3012	0.0022
13	0.2840	0.2725	-0.0115
14	0.2660	0.2466	-0.0194
15	0.2510	0.2231	-0.0279
16	0.2400	0.2019	-0,0381
17	0.2300	0.1827	-0.0473
18	0.2220	0.1653	-0.0567
19	0.2130	0.1496	-0.0634
20	0.2040	0.1353	-0.0687

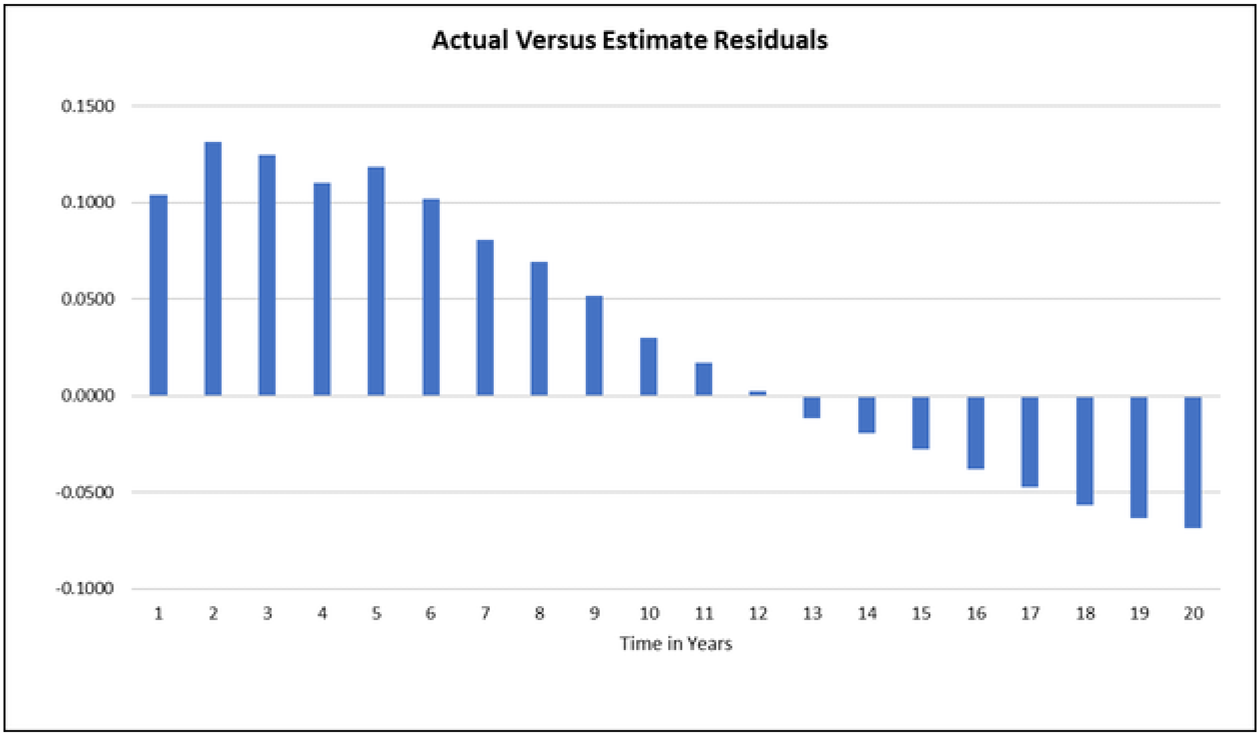
Table 4: Survival Rates Mapped To An Exponential Distribution



Note that in the graph above survival rates mapped to an Exponential distribution is not that accurate as survival rates are over-estimated over the first ten years and under-estimated over the last then years.

Residual Estimation via Linear Regression

Using the data in Table 3 above the graph of the residuals is...



Note that the residuals in the graph above is approximately linear and therefore we will use linear regression to estimate the equation for that line. The Least Squares Estimates (LSE) via the Excel regression tool (the independent variable is time and the dependent variable is the residual value) are...

Symbol	Description	Value
R2	R-squared	0.96511
α	Constant	0.15297
β	Beta coefficient	-0.01167

Using Equations (1) and (4) above and the data in the regression parameters table above the composite survival rate equation is...

$$S(t) = \text{Exp}\left\{ -\lambda\, t \right\} - \left(\alpha + \beta\, t \right) \;\text{...when...}\; t > 0 \tag{5}$$

Using Equation (5) above and the data in the regression parameters table above the composite equation for the expected survival rate at time t for our hypothetical problem is...

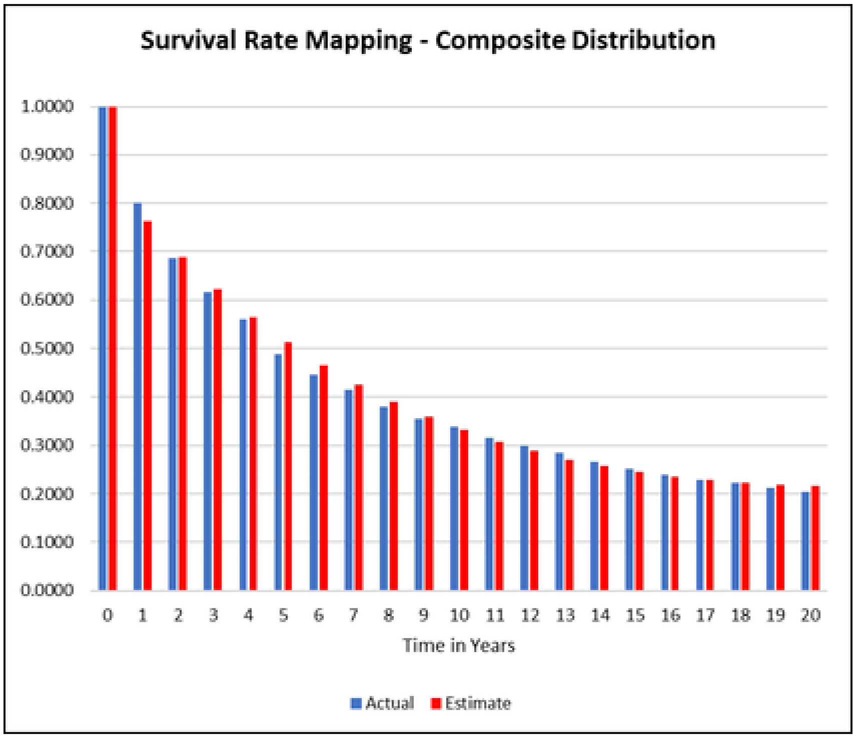
$$S(0) = 1.0000 \;\text{...when...}\; t = 0$$

$$S(t) = \text{Exp}\left\{ -0.1000 \times t \right\} - 0.15297 + 0.01167 \times t \;\text{...when...}\; t > 0 \tag{6}$$

Table 5: Survival Rates

Birth Year	Actual Rate	Composite Rate
0	1.0000	1.0000
1	0.8010	0.7635
2	0.6870	0.6891
3	0.6160	0.6229
4	0.5600	0.5640
5	0.4880	0.5119
6	0.4470	0.4659
7	0.4160	0.4253
8	0.3800	0.3897
9	0.3550	0.3586
10	0.3380	0.3316
11	0.3160	0.3083
12	0.2990	0.2883
13	0.2840	0.2713
14	0.2660	0.2570
15	0.2510	0.2452
16	0.2400	0.2357
17	0.2300	0.2281
18	0.2220	0.2224
19	0.2130	0.2183
20	0.2040	0.2158

Table 6: Survival Rates Mapped To A Composite Distribution



Note that in the graph above the composite survival rate equation (Equation (5) above) gives us a much better (almost perfect) estimate of actual JP Morgan survival rates.

The Answer To Our Hypothetical Problem

Question: Given that ABC Company has survived to year 3.25, what is the probability that ABC company will survive until year 7.75?

Using Equation (6) above the unconditional probability that ABC Company survives to year 3.25 is...

$$\mathrm{Prob}\left[\mbox{Survive until year 3.25}\right] = \mathrm{Exp}\left\{-0.1000 \times 3.25\right\} - 0.15297 + 0.01167 \times 3.25 = 0.6075 \tag{7}$$

Using Equation (6) above the unconditional probability that ABC Company survives to year 7.75 is...

$$\mathrm{Prob}\left[\mbox{Survive until year 7.75}\right] = \mathrm{Exp}\left\{-0.1000 \times 7.75\right\} - 0.15297 + 0.01167 \times 7.75 = 0.3982 \tag{8}$$

Using Equations (7) and (8) above the answer to the question is... [3]

$$\mathrm{Prob}\left[\mbox{Survive until year 7.75}\,\Big|\,\mbox{Survived to year 3.25}\right] = \frac{0.3982}{0.6075} = 0.6555 \tag{9}$$

References

[1] Gary Schurman, *The Exponential Distribution - The Mathematics,* March, 2012.

[2] Gary Schurman, *Maximum Likelihood Estimation - The Exponential Distribution,* March, 2023.

[3] Gary Schurman, *Startup Survival Rates - Discrete-Time Survival Rate Curve,* March, 2023.